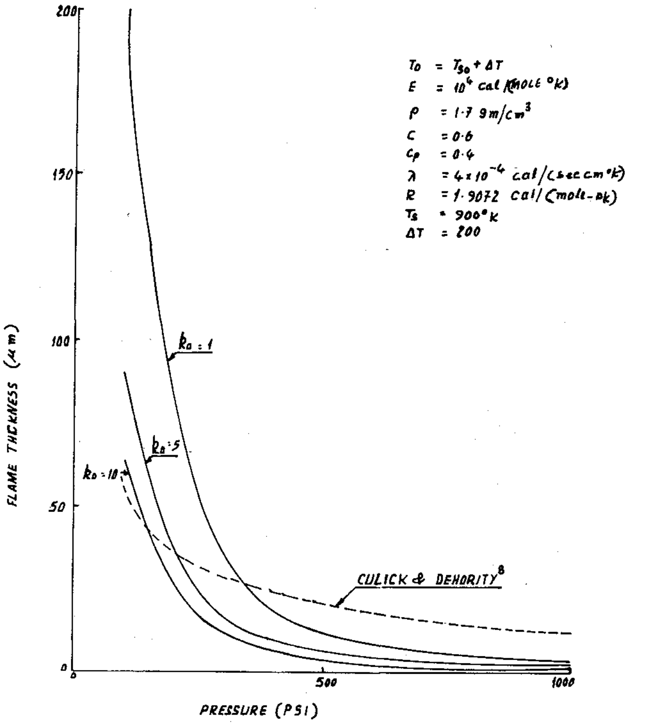

Fig. 1 Dependence of flame thickness on pressure and reaction constant.

$$\times \frac{\chi}{\lambda}\,(\omega\cos\omega\tau + \sigma\sin\omega\tau)\exp(-\sigma\tau)$$

$$\psi_2 = -\left[L + \frac{F_0}{\rho {r_0}^2}\,(T_f - T_{s0})\right]\frac{\chi}{c_p r_0}\sin\omega\tau\,\exp(-\sigma\tau)$$

$$+\frac{M}{(\sigma^2+\omega^2)}\exp(-\sigma\tau)\left[L - \frac{F_0}{\rho {\gamma_0}^2}\,(T_f - T_{s0})\right][\sigma\cos\omega\tau$$

$$-\omega\sin\omega\tau]\,\theta = \tfrac{1}{2}\tan^{-1}\left[4\frac{\omega\lambda}{\rho c_p {r_0}^2 + 4\sigma\lambda}\right],\ \beta = \left[\left(1 + 4\frac{\sigma\lambda}{\rho c_p {r_0}^2}\right)^2\right.$$

$$\left.+\frac{16\omega^2\lambda^2}{\rho^2 c_p^2 {r_0}^4}\right]^{1/4} \quad M = \frac{2\beta\sin\theta}{1+\beta^2 - 2\beta\cos\sigma}$$

Equation (23) gives the amplitude of the surface temperature variation and thus the small perturbation in surface temperature is given by

$$\Delta T_s(t) = \frac{F_1}{c_p {r_0}^2}\,(T_f - T_{s0})\,({\psi_1}^2 + {\psi_2}^2)^{-1/2}$$

$$\times\exp\left[i\left(\omega t + \tan^{-1}\frac{\psi_2}{\psi_1}\right)\right] \tag{24}$$

The positive real parts of the solution (24) indicate unstable behavior. To investigate stability limit, the real part of the solution is equated to zero, which gives

$$\psi_2 + \psi_1 \cot\omega t = 0 \tag{25}$$

Equation (25) gives the necessary condition for stability. By varying the parameters, the influences of changes in physical parameters of the composition on unstable burning can be predicted.

Conclusion

Perturbation technique has been employed to investigate the transient process of combustion of solid propellant. We have presented a discussion on quantities which are not measurable, for example, the flame thickness and chemical reaction rate. Variation of these quantities are shown graphically. The advantage of this analysis is that it enables one to determine quite easily how the various parameters influence the combustion mechanism.

Secondly, a range for stability in terms of known parameters is deduced which can help designers to predict the influence of changes in propellant's parameters on the unstable burning.

References

[1] Geckler, R. D., "Unsolved Problems in Solid Propellants Combustion," Fifth Symposium on Combustion, Reynold Publishing Co., New York, 1955, pp. 29-40.

[2] Green, L. Jr., "Observations on Irregular Reaction of Solid Propellant Charges," *Jet Propulsion*, Vol. 26, Aug. 1956, pp. 655-659.

[3] Green, L., Jr., "Some Properties of a Simplified Model of Solid Propellant Burning," *Jet Propulsion*, Vol. 28, June 1958, pp. 386-392.

[4] Rice, O. K. and Ginell, R., "The Theory of Burning of Double-Base Rocket Powders," *Journal of Physical Chemistry*, Vol. 54, 1950, pp. 885-917.

[5] Parr, R. G. and Grawford, B., Jr., "A Physical Theory of Burning of Double-Base Rocket Propellants," *Journal of Physical Chemistry*, Vol. 54, 1950, pp. 929-954.

[6] Sotter, J. G., "Chemical Kinetics of the Cordite Explosion Zone," Tenth Symposium (International) on Combustion of Solid Propellants, The Combustion Institute, 1965, p. 1405.

[7] Culick, F. E. C., "An Elementary Calculation of the Combustion of Solid Propellants," *Astronautica Acta*, Vol. 14, Feb. 1969, pp. 171-181.

[8] Culick, F. E. C., and Dehority, G. L., "An Elementary Calculation for the Burning Rate of Composite Solid Propellants," *Combustion Science and Technology*, Vol. 1, Sept. 1969, pp. 193-204.

[9] Aldushin, A. P. and Khaikin, B. I., "Effect of Reversibility of the Chemical Reactions on the Temperature and Pressure Dependence of the Burning Rate," *Combustion, Explosion and Shock Waves*, Vol. 7, (3), July-Sept. 1971, pp. 311-316.

[10] Schultz, R., Green, L., Jr. and Penner, S.S., *Studies of the Decomposition Mechanism, Erosive Burning, Sonance and Resonance for Solid Composite Propellants, Combustion and Propulsion*, in third Agrad Colloquium on Combustion & Propulsion, edited by Tring, M.W., Pergamon Press, New York, 1958, pp. 367-422.

Technical Comments

Comment on "Optimization of Ring Stiffened Cylindrical Shells"

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RELIABILITY is a major problem associated with nonlinear mathematical programming (NLP) optimization methods. Eason and Fenton[1] in a recent comparison study of some 17 NLP codes found that a typical NLP code solved only half of the ten test problems studies. Thus the user of such methods cannot assume that merely because he applies some popular NLP procedure to a given design problem it will generate optimal solutions. Confirmation studies and/or some check on the optimality of the solutions are needed to establish that the NLP method selected indeed works on the problem to which it has been applied. Any

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presentation of an optimal design procedure using an NLP algorithm should, therefore, address the reliability question if the potential user of the procedure is to be properly informed as to its performance.

The recent article by Bronowicki et al.[2] demonstrates the need for optimality confirmation and a discussion of solution reliability. The problems treated there are quite difficult in the NLP sense. Application of the DSDA (a procedure which appears reasonably reliable, solving 9 of 10 of the problems of Eason and Fenton)[3], to a problem very similar to the type 1 problem of Ref. 2 produced optimization algorithm failure. It was necessary to resort to variable coupling and mixed discrete-continuous programming techniques to obtain solution reliability.[4] SUMT procedures such as used in Ref. 2 appear to be substantially less reliable than DSDA. The best of the SUMT based procedures tested by Eason and Fenton solved only half of the test problems.[1] Thus, it seems reasonable to suspect reliability problems with such a procedure on this difficult shell problem.

The 1,000 ft study for problem type 1 of Ref. 2 was checked by a new code using constraint equations, which are identical to those of Ref. 2, except for the hull vibration and buckling equations which are quite similar.[5] The shall parameter values of Ref. 2 were used in the study. A value for the weight/displacement ratio was obtained that is about 5% lower than that reported in Ref. 2 and some 19% lower than reported in the original report[6] on which Ref. 2 is based. It should be noted that the design reported in Ref. 2 cannot be considered typical of the performance of the optimization procedure used there since it was developed on the basis of information generated by another similar, shell optimization capability.[2,4] A difference of 5%, although perhaps inconsequential considering the accuracy of the behavior equations used, is significant from a mathematical programming viewpoint. Thus, the design for the 1,000 ft, type 1 problem in Ref. 2 does not appear to be near optimum in a mathematical programming sense.

Furthermore the design presented as optimal in Ref. 6, and as a local optimum in Ref. 2, is apparently neither. This design was used as a starting point for a synthesis run using the program of Ref. 5. The search immediately moved away from this point. Since the minimum natural frequency constraint and shell buckling constraints are not active for this design, and thus the active constraints equations for this point are identical to those of Ref. 2 and 6, one can conclude that the point is not a local optimum. Thus it appears the NLP procedure used in Ref. 2 simply failed at this point.

This writer concludes, on the basis of his studies,[5] earlier comparison studies,[1] the difficulty of the problem,[4] the differing results presented in Refs. 2 and 6, and the starting point sensitivity cited in Ref. 2, that the optimization procedure described therein is unsuitable for the problems posed.

Some additional, less important, points are also noteworthy. Although the limitation on the vibration mode search to the range $0 \leq n \leq 6, 1 \leq m \leq 6$ does not affect any of the results presented, it should be noted that the use of this limitation could produce invalid designs for the parameters used in the 1,000 ft case if a minimum natural frequency of zero is specified since an optimal design with these parameters will have both a boundary ($m=1$) and interior ($m \geq 6$) buckling minimum active.[7]

The elimination of the inconsistency of using a static rather than dynamic constraint for the interring shell mode can be effected without difficulty by applying the frequency equations of Ref. 2 to the interring shell segment. The treatment of modes shapes for the interring shell panel would not be inconsistent with the use of an orthotropic shell model[2] since the panel is unstiffened and thus the model reduces to the isotropic case where the limitations cited in Ref. 2 do not apply.

Considering the form of the objective function and parameters used in the paper, the procedure appears intended for the design of shells submersed in water. Yet at least for some of the paramenters studied, the in-vacuo frequencies such as those employed in Ref. 2 are a poor approximation to the frequencies of immersed shells. Studies of a stiffened shell with R/t_s and L/R ratios very close to those of the 1,000 ft case show that the lowest frequencies in water are about half of those in air.[8] Thus, it seems unrealistic to specify a minimum in-vacuo frequency when the actual frequencies may be much lower. Furthermore it seems inappropriate to employ NLP methods in a situation where one cannot obtain a reasonable estimate of the merit of a design. Thus the applicability of problem types 2 and 3 to submerged shells seems questionalble, since in-vacuo frequencies are used in the objective function of these problems.

Finally the differences between the weights of the designs presented in Ref. 2 and Ref. 4 are not due in any significant degree to variable coupling as stated in Ref. 2. Rather they are due primarily to the inclusion in Ref. 4 of a stiffener buckling constraint,[9] which is apparently unjustified and not used in Ref. 2.

References

[1]Eason, E.D. and Fenton, R.G., "A Comparison of Numerical Optimization Methods for Engineering Design," *Journal of Engineering for Industry, Transactions of the ASME,* Vol. 96, Ser. B, Feb. 1974, pp. 196-201.

[2]Bronowicki, A.J., Nelson, R.B., Felton, L.P., and Schmit, L.A., Jr., "Optimization of Ring Stiffened Cylindrical Shells," *AIAA Journal,* Vol. 10, Oct. 1975, pp. 1319-1325.

[3]Pappas, M., "Performance of the 'Direct Search Design Algorithm' as a Numerical Optimization Procedure," *AIAA Journal,* Vol. 13, June 1975, pp. 827-829.

[4]Pappaer, M. and Allentuch, A., "Automated Optimal Design of Frame Reinforced Submersible, Circular, Cylindrical Shells," *Journal of Ship Research,* Vol. 17, Dec. 1973, pp. 208-216.

[5]Pappas, M., "Improved Synthesis Capability for "T" Ring-Stiffened Cylindrival Shells Under Hydrostatic Pressure," Presented at the Second National Symposium on Computerized Structural Analysis and Design held in Washington, D.C. on March 29-31, 1976, to be published in *Computer and Structures.*

[6]Bronowicki, A., Nelson, R.B., Felton, L.P., and Schmit, L.A., Jr., "Optimum Design of Ring Stiffened Cylindrical Shells," School of Engineering and Applied Science, UCLA, Los Angeles, Calif. UCLA-ENG-7414, Feb. 1974.

[7]Pappas, M. and Allentuch, A., "Pressure Hull Optimization Using General Instability Equations Admitting More Than One Longitudinal Buckling Half-Wave," *Journal of Ship Research,* Vol. 19, March 1975, pp. 18-22.

[8]Paslay, P.R., Tatge, R.B., Wernick, R.J., and Muster, D.F., "Vibrational Characteristics of a Submerged Ring-Stiffened Cylindrical Shell of Finite Length," *The Journal of the Acoustical Soceity of America,* Vol. 46, 1969, pp. 701-710.

[9]*Principles of Naval Architecture,* Society of Naval Architects and Marine Engineers, N.Y., 1967.

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PAPPAS comments on two main points regarding the optimization study in Ref. 1, namely that: 1) the numerical

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